Experiment 04

Determination of speed and acceleration of a toy car

By

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1.) Hypothesis

Time and distance are the most principle constituents of describing motion in an everyday life context. The average speed of a body is the distance (s) it travels per time (t). In mathematical terms it is expressed as in the following equation:

\[ \text{Speed} = \frac{s}{t} \]

Does the distance (numerator) increase and the time (denominator) stays the same, the distance travelled in the same time period is bigger and therefore the average speed increases. Does the time in which a body moved over a certain distance decrease, the average speed of that body increases.

However, distance is a scalar that means it is a mere numerical value and does not consider the direction in which the distance was travelled. This concept is visualised in figure 1. The distance between 0 and P is the same as the distance 0 and P'. However, if the distance is seen from a reference point (0), the distances are in opposite directions. In the concept of a distance, due to this size being a scalar, this difference in direction cannot be expressed. Therefore the concept of displacement was introduced. This size is a vector and therefore includes a direction in its value. The distance travelled in the concept of displacement always refers to a point from which it is measured, i.e. where the displacement is zero. The direction in the concept of displacement is expressed by the algebraic sign of the value. The direction which is positive and the direction which is negative have to be determined before considering any problematic in which displacement is involved. When it is looked at figure 1 again for example it could be said the direction to the right of 0 was positive, and the direction to the left of 0 was negative. Hence, the displacement from 0 to P would be a positive value, and the displacement 0 to P' would be a negative value while a displacement from P to 0 would be given by a negative value, and a displacement from P' to 0 would be given by a positive value. The sum of displacement of 0 to P and P to 0 would be zero; the same is true for the sum of the displacement from 0 to P' and the displacement from P' to 0. In the concept of distance the sums were not zero, they were two times the distance from 0 to P respectively 0 to P', because distance does not refers to a point and therefore is always described by a positive value.

The average speed is also a scalar due to distance and time being a scalar as well; it does not consider in which direction a body is moving. However, when the concept of distance in the equation above which defines speed is substituted by the concept of displacement the movement described takes the direction in which the movement occurs into account, and
therefore is a vector. This vector size is described by the concept of velocity. Velocity is mathematically defined as shown in the following:

$$v = \frac{d}{t}$$

Where \(v\) is the average velocity and \(d\) is the displacement. The unit of the velocity is metres per seconds (ms\(^{-1}\)).

However, the principle concept of average velocity is a very crude way of describing motion. It does not take into account any changes in velocity during the movement from one point to another. To describe a motion more accurately the whole motion can be subdivided into sub-movements. Each of those sub-motions can be considered as being a motion in its own right. This is done by splitting the whole displacement involved in the motion into several pieces. The actual displacement of each of those pieces can be determined by subtracting the end point of the piece by the start point of the piece. The change in displacement is defined as \(\Delta d\) (\(\Delta\): delta) as in the following:

$$\Delta d = \text{final displacement} - \text{initial displacement}$$

Figure 2 visualises how a motion can be divided into several “motion pieces”. The displacement of each of those pieces can be determined with formula for \(\Delta d\) as described above. The same principle can be used for time as well.

If the motion is a motion with constant velocity, i.e. the velocity does not change, and the intervals of the motion taken were taken in regularly distances from each other and have the same value of displacement, \(\Delta v\) of all intervals is the same. This relationship can also be expressed in the form of a displacement (y-axis)- time (x-axis) graph as shown in figure 3.
The gradient of the straight line is the velocity of the motion. Due to the graph being a straight line the velocity of that movement is constant (i.e. the value of the gradient). The velocity of movement described in figure 3 has therefore the value 1 m\(\text{s}^{-1}\).

If a body moves with a changing or non-uniform velocity, the graph does not look as simple as the graph showing a constant velocity. The values of \(\Delta\) velocity are not the same if the displacement pieces are chosen regularly. Those values of \(\Delta\) velocity give the average velocity between the two points involved. Therefore the velocity of the movement must change, which means it must be accelerated respectively decelerated. The displacement-time graph of a uniformly accelerated motion is shown in figure 4.
Because the motion is accelerated, the velocity of the body which moves changes. However, knowing that the gradient of the curve of displacement-time graph is always the velocity, the velocities for each point of the curve can be determined. Therefore a tangent (a straight line which touches the point in consideration) of the point in consideration can be drawn; afterwards the gradient of the tangent is measured; this value is the velocity at that point.

The average acceleration ($a$) of a body is the velocity change ($v$) per time period ($t$). This is mathematically expressed by the following equation:

$$a = \frac{\Delta v}{\Delta t}$$

Does the velocity of a movement of a body between two fixed points increase in the same time period, does the acceleration increase as well.

A graphical representation of the formula for the acceleration is a velocity (y-axis)-time (x-axis) graph. Figure 5 shows an example of such a graph.

![Figure 5: Velocity-time graph (constant acceleration)](image)

Figure 5 shows an example graph of a uniformly accelerated movement. The gradient of the line represents the acceleration of the movement. The constant acceleration and the average acceleration of a movement of a body can be assumed to be equal.

An example of a graph describing a non-uniformly accelerated movement is shown in figure 6.
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Figure 6: Velocity-time graph with non-uniform acceleration

The acceleration of one chosen point can be determined in the same way as the velocity of a chosen point in a displacement-time graph.

If the movement is non-uniformly accelerated, it might be useful to draw an acceleration-time graph.

With the help of the first two introduced types of graphs it is even possible to derive the acceleration of a movement from a displacement-time graph which describes the movement. The gradient(s) of the curve of the displacement-time graph represent the velocity of the movement. Therefore the velocity at some chosen points on the graph can be determined. With the help of those values in turn a velocity-time graph can be drawn. The gradient(s) of the curve of that graph represent the acceleration of the movement. If the movement had a constant velocity, the graph velocity-time graph would show a straight horizontally line, which means the gradient of the curve and therefore the acceleration of the movement is zero. If the movement is uniformly accelerated, the value of the acceleration can easily be determined because the resulting velocity-time graph shows a straight increasing/decreasing line. If it is not constant, the acceleration can be determined by the “tangent-method”. An acceleration-time graph might be drawn.

In this experiment the non-uniformly accelerated movement of a toy car is measured in a way that it makes it possible to draw all three graphs for the movement described above. Thereby the whole motion from rest to rest of the car is included in the analysis respectively the graph. Therefore it is not sufficient to measure the time and distance of the whole movement, because this would only give the average velocity of the whole movement. The experiment in contrast aims to deliver a more accurate description of the movement in terms of velocity and acceleration. Therefore the velocities between several sub-distances have to be measured, just as in the concept of $\Delta d$ shown above. This is realised by a method using the frequency of the national grid as a time measurement. To carry out the time measurement a long thin strip of paper (ticker timer) is fixed to a “pull-back” toy car. Afterwards the ticker tape is tailored in
the opening of a ticker tape timer. At the same time the ticker tape timer is connected to the national grid. A ticker tape timer is a device that makes a point on the ticker tape every time a whole wave of an alternating current passes through it. The frequency used in the experiment is the frequency of the national grid, which is 50 Hz. That means 50 waves per second pass through the ticker tape timer; therefore it makes 50 points on the ticker tape in one second.

When the car is “pulled back” and released, the car and the ticker tape start to move. Thereby the ticker tape timer is making points on the moving ticker tape every 1/50 s. Because the car is moving the points will be in a sequential order on the tape. The higher the velocity of the car the further apart will the two points be apart from each other, because the time period between two points is constant (1/50 s). Therefore, the distance moved during a time period can be obtained from the ticker tape recordings. Therefore the velocity between the points can be calculated. The acceleration between the two points can therefore afterwards be calculated as well. With the obtained values the three graphs can be drawn.

In view of the car starting from rest and ending in rest, due to Newton’s first law of motion there must be forces involved in the movement. Firstly, the power of the mechanics inside the car releases the energy which was given to it by “pulling” the car back. Therefore the car accelerates, which in turns means the velocity of the car increases. However, at some point the car’s velocity decreases again; this means there must be another force involved which is friction.

During the movement the energy release of the car mechanics decreases, therefore the acceleration slightly decreases, while the velocity still increases. This occurs until the point is reached at which the decelerating friction force is greater than the accelerating force of the mechanics. From this point on the velocity falls again, while the acceleration becomes negative. At some point all the energy in the mechanics is used. Therefore the only accelerating (i.e. decelerating) force is friction. The velocity decreases until it becomes zero. In view of friction being proportional to velocity, the acceleration should reach a negative maximum point and then gradually goes against zero. It is zero when the car is at rest again.
2.) Diagram

Figure 7: Set-up of the experiment to determine velocity and acceleration of the motion a toy car

3.) Method

The principle set-up of the experiment is shown in figure 7. A length of ticker tape was attached to the “pull-back car with tape. The length of the ticker tape was longer than the distance the car was expected to be moving. To get an impression of the movement character of the car, the car was pulled back a measured distance without ticker tape measurement. This distance was also used during the measurements.

For the actual measurement the ticker tape was tailored through the gap in the ticker tape timer. The car was pulled back the same distance and with the same pressure on it as described above. Before it was released, the power unit had been switched on. Therefore an electrical current with the frequency 50 Hz was flowing through the ticker tape timer. Afterwards the car was released along the bench. It was not interfered with movement until the car came to rest.

The ticker tape time made a point onto the ticker tape every 1/50 second. The length from the first point on the ticker tape to the last point on the ticker tape was measured with a ruler. Subsequently, the points were counted and sub-divided into “eleven point” packs. The end point of such a pack was the starting point of next pack at the same time (i.e. point 1-11, point 11-22, etc.). Therefore, there were ten gaps in such an eleven point pack. The distance of each of those packs was measured with a ruler. The last pack did not contain eleven points but only five points (i.e. 4 gaps). The distance of this last incomplete pack was measured as well.

At the very end of the tape the gaps between the points were so small that they appeared as a line. Those points were not included in the “eleven packs”.

The same procedure was repeated two times to obtain three independent measurements of the movement of the car.
4.) Errors

4.1) Systematic errors
➢ The smallest sub-division of the ruler used for all length measurements involved in the experiment was 1 mm. Therefore a systematic error of +/- 0.5 mm occurred.

4.2) Reading errors
➢ A parallax error occurred during all length measurements, due to the angle at which the value was read.

4.3) Random errors
➢ At the very end of the ticker tape the gaps between the points were so small that they appeared as being one single line. Those values were not considered in the analysis.
➢ The car did not move exactly in a straight line. Some points on the ticker tape therefore did not lie on a straight line. Therefore the measured distance was slightly smaller than the actual distance.
➢ It was assumed that the average velocity between two points was the actual velocity of the second point.

5.) Results
The aim of the experiment was to draw three graphs, namely a displacement-time graph a velocity-time graph and an acceleration time graph. Three readings were carried out. The measured and calculated results which were used to plot three times three graphs are found in table 1 to 3. The following paragraphs show how the three graphs were obtained for one reading. This reading is the first reading; the corresponding values are shown table 1. The graphs of the other two readings were obtained in the same principle way with the corresponding values obtained from the experiment for the two readings (Table 2 and 3).

Firstly, a displacement-time graph had to be drawn. Therefore both, time values and corresponding displacement values needed to be measured.

The time between two points on the ticker tape was 1/50 s due to the frequency of the national grid. The distance between two points is the distance the car travelled in 1/50 s. The distance could have been measured with a ruler. Therefore the values for time periods (1/50 s) and the corresponding displacement values could have been obtained from the ticker tape by measuring the distances between each two neighbouring points. Therefore the velocities between those points could have been calculated.

However, for convenience the distance between eleven points were measured. The time period between eleven points was 10/50 s, due to the fact that ten gaps (ten 1/50 s periods) lay between eleven points. Therefore the time period was 10/50 s for each “eleven point pack”. The distances between each eleven points is therefore the distance the car moved in 10/50 s. To obtain suitable values for plotting a displacement-time graph of the movement,
the distances of the eleven point packs were added to each other in the sequence they were on the ticker tape (in the direction of the movement); each of those values were noted (see table 1). Afterwards the same was done for the corresponding time values (see table 1). Therefore all values were obtained to draw the displacement-time graph. It is shown in figure 8.

With the help of the actually measured values, the average velocities between each eleven gaps could be calculated. This was done in the following way. This calculation should serve as an example calculation; it shows how the average velocity from 1) the starting point to point eleven and 2) from point 11 to point 22 of the first measurement (table 1) was calculated:

1)  
\[ v = \frac{\Delta d}{\Delta t} \]  
\[ v = \frac{x}{10/50} \]

\[ v = \frac{0.025}{0.05} = 0.125 \text{ ms}^{-1} \]

The average velocity between the starting point and point eleven of the first reading was therefore 0.125 ms\(^{-1}\).

2)  
\[ v = \frac{\Delta d}{\Delta t} \]  
\[ v = \frac{x}{10/50} \]

\[ v = \frac{0.062}{0.05} = 0.31 \text{ ms}^{-1} \]

The average velocity between point 11 and point 22 of the first reading was therefore 0.31 ms\(^{-1}\).
The same calculation with the corresponding values for time and length were carried out for the other “eleven point packs” of the reading. Therefore several values for the velocity at different points of the movement were obtained.

With the help of those values a velocity-time graph was plotted. It is shown in figure 9.

The last pack of points at the end of the ticker tape did in the first and in the third reading not consist of 10 but 4 gaps. The principle calculations for velocity and acceleration were the same as for the other points; only the time period had to be changed into 4/50.

In the next step, the accelerations between the points were determined. Thereby it was assumed that the average velocity between two points was the actual velocity of the second point. To gain $\Delta v$ between eleven points the velocity at the second point ($v$) was subtracted by the velocity at the first point ($u$). This value was then divided by the time which passed between the 11 points (10/50s). In the following it is shown how the calculation with which the average acceleration between point 11 and point 22 of the movement which took place in the first reading was carried out:

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v - u}{\frac{10}{50}}$$

$$a = \frac{0.31 - 0.125}{\frac{10}{50}} = 0.625 \text{ ms}^{-2}$$

The average velocity between point 11 and point 22 in the first reading was therefore 0.625 ms$^{-2}$.

The same principle calculation was carried out for the other values as well. With those values an acceleration-time graph was drawn. It is shown in figure 10.

The same procedure as described in the paragraphs above was carried out for the values of the two other readings. The data is shown in table 2 and 3, while the graphs are shown in figure 11 to 16.
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<table>
<thead>
<tr>
<th>Overall distance (m)</th>
<th>Distance (10 gaps) (x/m)</th>
<th>Time (t/s)</th>
<th>Velocity (v/ms(^{-1}))</th>
<th>Acceleration (a/ms(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.025</td>
<td>0.2</td>
<td>0.125</td>
<td>0.625</td>
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<td>0.087</td>
<td>0.062</td>
<td>0.4</td>
<td>0.31</td>
<td>0.925</td>
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<td>1.2</td>
<td>0.7125</td>
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<td>1.4</td>
<td>0.59</td>
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<td>1.6</td>
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<td>1.8</td>
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<td>0.0375</td>
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Table 1: Data table for reading 1

<table>
<thead>
<tr>
<th>Overall distance (m)</th>
<th>Distance (10 gaps) (x/m)</th>
<th>Time (t/s)</th>
<th>Velocity (v/ms(^{-1}))</th>
<th>Acceleration (a/ms(^{-2}))</th>
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<td>3.4</td>
<td>0.034</td>
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<td>0.17</td>
<td>0.85</td>
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<tr>
<td>10.4</td>
<td>0.07</td>
<td>0.4</td>
<td>0.35</td>
<td>0.9</td>
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<td>20.9</td>
<td>0.105</td>
<td>0.6</td>
<td>0.525</td>
<td>0.875</td>
</tr>
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<td>34.4</td>
<td>0.135</td>
<td>0.8</td>
<td>0.675</td>
<td>0.75</td>
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<td>49.6</td>
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<td>1</td>
<td>0.76</td>
<td>0.425</td>
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<tr>
<td>63.2</td>
<td>0.136</td>
<td>1.2</td>
<td>0.68</td>
<td>-0.4</td>
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<td>74.3</td>
<td>0.111</td>
<td>1.4</td>
<td>0.555</td>
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<td>82.3</td>
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Table 2: Data table for reading 2

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<th>Distance (10 gaps) (x/m)</th>
<th>Time (t/s)</th>
<th>Velocity (v/ms(^{-1}))</th>
<th>Acceleration (a/ms(^{-2}))</th>
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<td>0.68</td>
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<td>0.075</td>
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Table 3: Data table for reading 3
6.) Conclusion/Evaluation

In view of the graphs shown in figure 8 to 16, the graphs of the first reading seem to match best with the expected form of the graphs. Especially the third reading seemed unlikely to be accurate. The deceleration to the end of the movement steadily increases instead of going slowly against zero.

The obtained values for velocity and acceleration seemed reasonably small. The whole length of the movement was only one metre for an object of around 15 cm length. The driving power of the car was also not very strong compared to its mass. Therefore no big values for either velocity or acceleration could be expected. On the other hand, in a 100 m sprint for example the distance compared to the length of the movement is much longer and the driving power is much bigger compared to the moved object. Top sprinters can reach an average velocity of up to 10 ms\(^{-1}\), which is much faster than the car in the experiment.

The method used in the experiment did not seem to work for the “small number of points packs” at the end of the measurement. The obtained values for velocity and acceleration were far from being reasonable. They did not fit into the line. This may be due to the fact that the average velocities between two points were considered to be the actual velocity at the second point. The difference between the real actual value and the assumed actual value is bigger if the next pack of points in the row does only have 4 instead of 11 points.

7.) Bibliography

7.1 Notes)

7.2) Figures
Figure 1: Adapted from: Tsokos, K. A.; Physics for IB Diploma. (2005). Cambridge University Press.

7.3) References